

NUMERICAL ANALYSIS OF ELASTOHYDRODYNAMIC LUBRICATION WITH  
BIO-BASED FLUIDS

DEDI ROSA PUTRA CUPU

UNIVERSITI TEKNOLOGI MALAYSIA

NUMERICAL ANALYSIS OF ELASTOHYDRODYNAMIC LUBRICATION  
WITH BIO-BASED FLUIDS

DEDI ROSA PUTRA CUPU

A thesis submitted in fulfillment of the  
requirements for the award of degree of  
Master of Engineering (Mechanical)

Faculty of Mechanical Engineering

Universiti Teknologi Malaysia

NOVEMBER 2012

*To my beloved parents, my siblings & friends.*

## ACKNOWLEDGEMENT

With the grace of Allah, the Almighty, the most gracious and merciful, finally I was able to complete this project research.

I would like to express my sincere appreciation to my supervisors, Dr. Jamaluddin Md. Sheriff and Assoc. Prof. Dr. Kahar bin Osman for the continuous encouragements, guidance, advices, and criticisms throughout this research. Without their supports, this thesis would have not been the same as presented here.

I would also like to thank to Mr. Zubil Bahak for his suggestion, sharing the expert knowledge in lubrication theory and improvement in this thesis, and for all people of the Computational Fluid Dynamic (CFM) lab of Universiti Teknologi Malaysia (UTM) for the real supports, discussions, and for our thought-provoking conversations. I also appreciate to all researchers in the tribology field, who have shared their knowledge, and to all of my dear friends and fellow colleagues and others who have provided assistance and support on various occasions.

Finally, my grateful thanks are presented to my parents, brothers and sisters who encouraged me to complete my study by finishing this research. This thesis is dedicated to you, my family.

## ABSTRACT

During the last couple of decades, the level of public considerations of increasing world energy crisis and environmental issues in various industrial applications has risen, including in the application of lubricants in machine elements. In this study, a numerical approach was developed to investigate the feasibility to use vegetable oils as lubricants in application of roller element bearing, namely elastohydrodynamic lubrication (EHL), especially for the contact between the inner ring and the cylindrical roller element. This simulation solved Reynolds equation simultaneously with elastic deformation and pressure-viscosity equation to analyse EHL pressure and film thickness. In this simulation, some vegetable oils were used as lubricants and results were compared with mineral oils and synthetic oils that are available in the market. It was discovered that in the condition of  $W = 2.0452 \times 10^{-05}$ ,  $U = 1.0 \times 10^{-11}$ , and  $T = 40^{\circ}\text{C}$ , camellia oil was the best vegetable oil to replace mineral oil or synthetic oil because the maximum pressure working on the contacted surfaces of roller element bearing was lower than those of other vegetable oils. However, all simulated vegetable oils can be used as lubricants based on their pressure profiles and film thicknesses. The effects of some parameters, such as applied load, speed and temperature on the pressure distributions and film thickness profiles were also studied for all vegetable oils. The results demonstrated that the pressure and film thickness increased as the speed and load increased, but the increase of the temperature caused the pressure and film thickness to decrease.

## ABSTRAK

Dalam beberapa dekad yang lalu, peringkat pertimbangan awam kepada peningkatan krisis tenaga dan isu-isu pada alam sekitar dalam pelbagai aplikasi perindustrian telah meningkat, termasuk penggunaan pelincir dalam elemen mesin. Dalam kajian ini, pendekatan berangka telah dibentuk untuk menyiasat kemungkinan menggunakan minyak sayuran sebagai pelincir dalam pemakaian gelas pengguling, iaitu pelinciran elastohydrodynamic (EHL), terutamanya untuk sentuhan antara cincin dalaman dan roller silinder. Simulasi ini menyelesaikan persamaan Reynolds serentak dengan persamaan elastik deformasi dan persamaan tekanan-kelikatan bagi mengira tekanan dan ketebalan filem. Dalam simulasi ini, beberapa minyak sayuran telah digunakan sebagai pelincir dan hasil kajian dibandingkan dengan minyak mineral dan minyak sintetik yang tersedia di pasaran. Dalam keadaan  $W = 2.0452 \times 10^{-05}$ ,  $U = 1.0 \times 10^{-11}$ , and  $T = 40^\circ\text{C}$  dalam kajian ini mendapati, minyak *camellia* ialah minyak sayur yang terbaik untuk menggantikan minyak mineral atau minyak sintetik kerana puncak tekanan yang bekerja pada permukaan gelas adalah lebih rendah daripada minyak sayuran lain. Walau bagaimanapun, semua minyak sayuran boleh digunakan sebagai pelincir yang berdasarkan profil tekanan dan bentuk ketebalan filem. Tambahan pula, kesan daripada beberapa parameter, seperti halaju, beban dan suhu ke atas tekanan dan ketebalan filem telah dikaji untuk semua minyak. Hasil kajian menunjukkan bahawa tekanan dan ketebalan filem meningkat kerana kelajuan dan beban meningkat, tetapi peningkatan suhu menyebabkan tekanan dan ketebalan filem menurun.

## TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENTS	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	x
	LIST OF FIGURES	xii
	LIST OF SYMBOLS	xv
	LIST OF APPENDICES	xvii
<b>1</b>	<b>INTRODUCTION</b>	<b>1</b>
	1.1 Background	1
	1.1.1 Fluid film lubrication	2
	1.1.2 Bio-based lubrication	3
	1.2 Objective and scope of study	6
<b>2</b>	<b>LITERATURE REVIEW</b>	<b>8</b>
	2.1 Line contact of elastohydrodynamic lubrication	8
	2.2 Bio-based lubricants	20
	2.3 Summary of study of bio-based lubricants	23

<b>3</b>	<b>MATHEMATICAL FORMULATION</b>	<b>26</b>
3.1	Initial parameters and properties of lubricants	27
3.2	Reynolds equation for Newtonian fluid	29
3.2.1	Equilibrium of forces on a lubricant element	30
3.2.2	Velocity distribution	34
3.2.3	Continuity of flow in a column	35
3.3	Reynolds equation for non-Newtonian fluid	39
3.4	Film thickness equation	40
3.4.1	Calculation of film thickness for steady state condition	41
3.5	Viscosity-pressure relations	45
3.6	Density-pressure relations	47
<b>4</b>	<b>NUMERICAL SOLUTION</b>	<b>48</b>
4.1	Introduction	48
4.2	Newton-Raphson method for solving Reynolds Equation	50
4.3	Numerical solution of elastohydrodynamic lubrication	52
<b>5</b>	<b>RESULT AND DISCUSSION</b>	<b>56</b>
5.1	Validation of program	56
5.2	Steady state EHL result	61
5.3	Effect of speed on the parameters of EHL	70
5.4	Loads effects on the parameters of EHL	74
5.5	Temperature effects on the parameters of EHL	77
<b>6</b>	<b>CONCLUSION</b>	<b>86</b>
6.1	Conclusion	86
6.2	Recommendation for future research	87
	<b>REFERENCES</b>	<b>89</b>



## LIST OF TABLES

TABLE NO.	TITLE	PAGE
2.1	Properties of oils by Ohno <i>et al.</i> (1997)	20
2.2	Properties of oils by Mia <i>et al.</i> (2007)	22
2.3	Physicochemical properties of soybean oil and sunflower oil	22
2.4	Summary of properties of tested oils	25
5.1	Pressure working on the roller element bearing obtained from Safa <i>et. al.</i> (1982) and the present study	59
5.2	Parameters of EHL for all simulated oils at temperature of 40°C	69
5.3	Effect of speed ( $u$ ) on pressure and film thickness	73
5.4	Effect of load ( $w$ ) on pressure and film thickness	76
5.5	Summary of the effect of load ( $w$ ) and speed ( $u$ ) on the pressure spike and film thickness using camellia oil.	76
5.6	Lubricant properties of rapeseed oil at various temperature	79
5.7	Pressure maximum at the centre of bearing ( $p_c$ ), pressure spike ( $p_s$ ) and film thickness for rapeseed oil at various temperatures.	82
5.8	Pressure spike ( $p_s$ ) of simulated vegetable oils at various temperature ( $W = 2.0452 \times 10^{-5}$ ; $U = 1.0 \times 10^{-11}$ )	84

- 5.9 Minimum film thickness ( $h_{\min}$ ) of simulated vegetable oils at various temperature ( $W = 2.0452 \times 10^{-5}$ ;  $U = 1.0 \times 10^{-11}$ ) 84

## LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
2.1	Conformal surface as shown in journal bearing	9
2.2	Non-conformal surface as shown in rolling element bearing	10
2.3	Pressure and film thickness of line contact EHL	11
2.4	Pressure profiles and film shapes at iterations 0, 1, and 14. Barus' formula; $W = 2.0452 \times 10^{-5}$ ; $U = 1.0 \times 10^{-11}$ ; and $G = 5007$	14
2.5	Dimensionless pressure profile and film thickness profile for isoviscous and viscous solution	14
2.6	Pressure profiles and film shapes for various dimensionless loads. Roelands pressure-viscosity formula; $U = 1.0 \times 10^{-11}$ ; and $G = 5007$	16
2.7	Effect of the slide/roll ratio on the TEHL	17
2.8	Result study of thermal EHL line contact problem	19
2.9	Equipment for measurement used by Mia <i>et al.</i>	21
2.10	Viscosity-pressure relations at temperature of 40°C	23
3.1	Dimensions of cylindrical roller bearing	27
3.2	Line contact geometry: (a) two discs; (b) equivalent contact pressure distribution	28
3.3	Geometry of the contact between the equivalent cylinder and the flat plate	29
3.4	Flow characteristics as a function of shear rate stress	31

3.5	Forces on element of lubricant	32
3.6	Continuity of flow in a column	36
3.7	Geometry of EHL line contact	41
3.8	Elastic deformation at any point of $x$	42
4.1	Graphical depiction of the Newton-Raphson method	49
4.2	Flow diagram of this study	55
5.1	Validation results of EHL pressure profile with experimental result. (a). This simulation. (b) Experimental recorded with oil at a fixed velocity of 4 m/s and loads of $1.1 \times 10^5$ (Safa <i>et al.</i> , 1982)	57
5.2	Overlapping of the validation results of EHL pressure profile between this study and the experimental result by Safa <i>et al.</i>	58
5.3	Pressure (a) and film thickness (b) distribution using palm oil; $W = 2.0452 \times 10^{-5}$ ; $U = 1.0 \times 10^{-11}$	62
5.4	Pressure (a) and film thickness (b) distribution using paraffinic mineral oil; $W = 2.0452 \times 10^{-5}$ ; $U = 1.0 \times 10^{-11}$	64
5.5	Pressure (a) and film thickness (b) using castor oil as lubricant; $W = 2.0452 \times 10^{-5}$ ; $U = 5.0 \times 10^{-12}$	65
5.6	EHL pressure profiles of mineral oils and synthetic oils for comparison purpose	67
5.7	EHL film thickness profiles of mineral oils and synthetic oils for comparison purpose	68
5.8	Speed effect on the EHL pressure distribution using mustard oil as lubricant	71
5.9	Speed effect on the EHL film thickness distribution using mustard oil as lubricant	72
5.10	Load effect on the EHL parameters using camellia oil as the lubricant.	75
5.11	Thermal effect on the EHL pressure distribution using rapeseed oil as lubricant	80
5.12	Thermal effect on the EHL film thickness distribution using rapeseed oil as lubricant	81

5.13	Thermal effect on the EHL pressure using vegetable oils as lubricant	85
5.14	Thermal effect on the minimum film thickness using vegetable oils as lubricant	85

## LIST OF SYMBOLS

$a$	-	weighting factor used to define $dP/dX$ at node $i$
$b$	-	Semiwidth of Hertzian contact, $2R\sqrt{2W/\pi}$ , m
$C_j$	-	Weighting factor
$D_{ij}$	-	Influence coefficient
$E$	-	Modulus of elasticity
$E'$	-	Effective elastic modulus
$G$	-	Material parameter, $\alpha E'$
$H$	-	Dimensionless film thickness, $Rh/b^2$
$H_0$	-	Dimensionless central film thickness at $X=0$
$H_{\text{end}}$	-	Dimensionless film thickness at outlet boundary
$h$	-	Film thickness, m
$h_e$	-	Film thickness where $\partial p/\partial x = 0$ , m
$h_0$	-	Film thickness at $x = 0$ , m
$i, j$	-	nodes
$K$	-	Dimensionless sliding constant, $3\pi^2 U/4W^2$
$N_{\text{max}}$	-	Maximum number of nodes used in mesh
$P$	-	Dimensionless pressure, $p/p_h$
$P_s$	-	Dimensionless pressure spike

$p$	-	Pressure, Pa
$p_H$	-	Maximum Hertzian pressure, $E' b/4R = E' \sqrt{W/2\pi}$ , Pa
$R$	-	Equivalent radius of contact, m
$r$	-	Radius of surface, m
$U$	-	Dimensionless speed parameter, $\eta_0 u/E' R$
$u$	-	Average entrainment rolling speed, $(u_a + u_b)/2$ , m/s
$W$	-	Dimensionless load parameter, $w/E' R$
$w$	-	Applied load per unit length, N/m
$X$	-	Dimensionless distance, $x/b$
$X_{\text{end}}$	-	Dimensionless location of the outlet boundary
$x$	-	Distance along rolling direction, m
$Z$	-	Roelands parameter
$\alpha$	-	Pressure-viscosity coefficient, $\text{m}^2/\text{N}$
$\rho$	-	Lubricant density, $\text{kg}/\text{m}^3$
$\rho_0$	-	Density at atmospheric pressure, $\text{kg}/\text{m}^3$
$\bar{\rho}$	-	Dimensionless density, $\rho/\rho_0$
$\bar{\rho}_e$	-	Dimensionless density where $H = H_e$
$\eta$	-	Lubricant viscosity, $\text{Ns}/\text{m}^2$
$\eta_0$	-	Lubricant viscosity at atmospheric pressure, $\text{Ns}/\text{m}^2$
$\bar{\eta}$	-	Dimensionless viscosity, $\eta/\eta_0$
$\nu$	-	Poisson's ratio
$\delta$	-	Elastic deformation, m
$\bar{\delta}$	-	Dimensionless elastic deformation, $\delta R/b^2$

## Subscripts

$a$	-	Surface $a$
$b$	-	Surface $b$
$H$	-	Hertz
$i$	-	at node number $i$
$j$	-	at node number $j$
$out$	-	Outlet position
$in$	-	Inlet position



**LIST OF APPENDICES**

<b>APPENDIX</b>	<b>TITLE</b>	<b>PAGE</b>
A	EHL pressure and film thickness profiles of vegetable oils	99
B	Results of speed effects on the parameters of EHL	112
C	Results of thermal effects on the parameters of EHL	119

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Background**

The purpose of this study is to prepare a numerical modeling of elastohydrodynamic lubrication, hereinafter referred to EHL, in order to calculate pressure profiles and film thicknesses in line contact, using bio-based oils as lubricants. Furthermore, this simulation was also developed to investigate the influence of variation in load, speed or curvature radius that engenders the squeeze effect on the parameters of EHL line contact problem. Temperature effect on the characteristics of EHL is moreover investigated by running simulation at various temperatures.

### 1.1.1 Fluid Film Lubrication

Tribology is a field science and technology of friction, wear and lubrication due to relative motion of surface contacts with liquids, known as lubricants. It derived from the Greek word *tribos* for ‘rubbing’. The word “tribology” was formally introduced since the publication of the “Department of Education and Science Report” which issued by Peter Josh in 1966 (Khonsari and Booser, 2008) as a chairman of the British Ministry of State for Education and Science committee. The report also concluded that saving money could be reached by fully improving in design lubrication, friction and wear. This discipline science is not only about mechanical field, but also involving chemical and material technology. One of the purposes of tribology is to optimize bearing designs, lubricants and materials for bearings by studying the reduction of friction and wear characteristics to conserve energy, increase productivity and reduce maintenance process (Hamrock *et al.*, 2004; Khonsari and Booser, 2008).

The fundamental aspects of hydrodynamic lubrication were discovered and formulated by N. P. Petrov (1836-1920), B. Tower (1845-1904), and O. Reynolds (1842-1912), as mentioned by Pinkus (1987). They realized that the lubrication process was not caused by mechanical interaction between two solid surfaces, but it was engendered by the dynamic of fluid film between those surfaces. Nicolai Petrov was interested in the friction area, who published two postulates: first, viscosity is the most important property of fluid, instead of its density; and second, friction in a bearing is produced by viscous hearing involving its fluid film.

Elastohydrodynamic lubrication is one of the hydrodynamic lubrication, which involving physical interaction between the contacting bodies and the liquid of lubricant causes these contacting surfaces will be deformed elastically and the changes of viscosity with pressure play fundamental roles. The contacting surfaces in many engineering applications, for example, roller element bearings, gears, cams,

seals, etc., are non-conformal; therefore, the consequent contact areas are very small and the resultant pressures are greatly high (Houpert and Hamrock, 1986).

Based on their solid contacted bodies, EHL generally consists of two types of problems, line contact problems and point contact problems. Contact between two spherical balls and contact between ball and flat surface are represented as point contact problems. Cylindrical roller bearing is represented as line contact problems. In the line contact type, the rolling and load zones are angularly centered and rolling zone is smaller than the load zone (Laniado-Jacome *et al.*, 2010).

Significant differences between Hydrodynamic lubrication (HL) and Elastohydrodynamic lubrication (EHL) involve the added importance of material hardness, increase of viscosity under high pressure, and degree of geometric conformation of the contacting surfaces. According to the operating conditions, EHL problems can be classified as a steady state where all variables involved are the time-independent cases and unsteady state (transient) problems where all variables (such as loading, entrainment speed and the contact curvature radius) change constantly in time (Cioc, 2004).

### **1.1.2 Bio-based Lubrication**

According to a 2007 Freedonia report (Bremmer and Plonsker, 2008), total lubricant demand in the whole world is expected to be about 41.8 million metric tons, or about 13 billion gallons, where Asia/Pacific region will be the fastest growth. The world market is segmented by application area is: Engine oils – 48%, Process oils – 15.3%, Hydraulic oils – 10.2%, and all other – 26.5%. The considerations of

increasing world energy crisis and environmental issues, in some countries, several laws and regulations have been enacted to control the production, application, and disposal of lubricants. These regulations have been released to minimize health hazards and water hazards (Bartz, 1998). Because of these two reasons, there is a need to source out biodegradable lubricants with technical characteristics superior to those based on mineral oils. It already in use as lubricants for applications of chainsaw bar lubricants, drilling mud and oils, straight metalworking fluids, food-industry lubricants, open gear oils, biodegradable grease, hydraulic fluids, marine oils, and outboard engine lubricants, oils for water and underground pumps, rail flange lubricants, shock absorber lubricants, tractor oils, agricultural equipment lubricants, elevator oils, mould release oils, two-stroke engine lubricants, cold forward extrusion and so on (Erhan and Asadauskas, 2000, Simon *et al.*, 2011).

The purpose of lubrication is to control friction and wear and also to provide smooth running and a satisfactory life for machine elements. It separates surfaces in relative motion by interposing a third body that has a low resistance to shear. These lubricants are usually made by blending base oil and a special chemical additive. The base oil can be a variety of different materials; most of them are liquids (such as mineral oils, water, synthetic oils), but they may be solids (such as polytetrafluoroethylene, or PTFE) for use in dry bearings, grease used in rolling-element bearings, or gases (such as air) for use in gas bearings.

The lubricant is selected based on a number of important factors; physical properties, chemical properties, lubrication properties, environmental friendliness and cost. Physical properties of fluid lubricant are characterized by temperature and chemical properties are characterized by oxidation and radiation influences, both affected by temperature. Mineral oils have been more used than synthetic oils because of their properties and performance features, such as thermal stability, oxidation stability, and viscosity temperature behaviour, temperature range of application and radiation stability. Synthetic oils will be selected for lubricants because the required chemical or physical property cannot be obtained by mineral

oils or required quality of mineral oils does not meet the standard of synthetic oils (Rudnick, 2002).

Bio-based oils are found in the seed or fruit of various plants or animals. These materials are usually nontoxic and environmental friendly. Vegetable oil is one of the bio-based oils that manufactured using seed or fruit of plants. Comparing to mineral oil-based lubricating oils, vegetable-based lubricants are many readily biodegradable and renewable resources. Vegetable oils have to be extracted or expressed from the plant tissue in the “crude” form, which contains several minor components like steroids, pigments, waxes, etc. Generally vegetable oils contain a combination of saturated and unsaturated fatty acids, where these acid compositions have large influence to the physical and performance properties of these oils.

Lubricants based on vegetable oils are renewable and possess high biodegradability, high viscosity index, and excellent coefficient of friction and higher wear rate, possess good boundary lubricant (Adhvaryu and Erhan, 2002, Erhan and Asadauskas, 2000, Jayadas *et al.*, 2007, Mia *et al.*, 2007, Musa, 2009, Mia and Ohno, 2010, Syahrullail *et al.*, 2011).

Some researchers have studied the possibility of usage the vegetable oils for the industrial application. Adhvaryu and Erhan (2002) had improved performance of epoxidized soybean oil and modified high oleic soybean oil genetically to overcome the poor thermal and oxidative of soybean oil so it could be used as high-temperature lubricants. Wan Nik *et al.* (2005) suggested using some food grade oils, such as palm oil, sunflower oil, coconut oil, canola oil and corn oil for hydraulic fluid. Jayadas and Nair (2006) reported that coconut oil is able to be used as base oil for industrial lubricants by modifying its thermal, oxidative and low-temperature properties.

Research in considering real measuring or testing the physical properties and tribological behaviour for vegetable oils as lubricants have been done (Ohno *et al.*, 1997, Mia *et al.*, 1997, Adhvaryu and Erhan, 2002, Wan Nik *et al.*, 2005). These

experimental concepts take a long time and need to destruct materials. However, at this moment, researchers need the accurate results quickly, and therefore study about bio-based lubricants used in line contact of elastohydrodynamic lubrication must be conducted and then a new numerical concept should be developed for this problem.

## 1.2 Objective and Scope of Study

Recently, the numerical solutions for EHL problems have been developed by many researchers, including for transient EHL of line contact. However, there are only a few of them used bio-based oils for their simulations, and therefore, the numerical method should be developed using bio-based oils in order to investigate a possibility to replace mineral oils as lubricants with the bio-based oils. Beside that, the effects of temperature on the parameters of EHL line contact are also investigated in this study by running the simulation in various temperatures.

The scopes for this project include:

- i. Numerical analysis is conducted to the cylindrical roller element bearing only; it means that solution is limited to the two-dimensional line contact problem.
- ii. Dimensionless load ( $W$ ) and dimensionless speed ( $U$ ) are set fixed as the paper of Houpert and Hamrock (1986) where  $W = 2.0452 \times 10^{-5}$  and  $U = 1.0 \times 10^{-11}$ . However, in order to investigate the effect of speed on the pressure distribution and film thickness profile, the average rolling speed ( $u$ ) of roller element is set varied between 10 mm/s and 750 mm/s. Then, the load effect on the EHL parameters is investigated by varying the applied load between 10 and 40 kN/m.

- iii. Temperatures are set at 0°C, 20°C, 40°C, 60°C, 80°C, and 100°C to show the thermal effect on the pressure and film thickness of EHL.
- iv. Some vegetable oils are chosen as the lubricants based on their viscosity index (*VI*) ranging from 75 to 200. According to Khonsari and Booser (2008), this range of *VI* is acceptable for industrial application. It should be noted that the viscosity index (and other properties) of these vegetable oils are obtained from other researchers' testing (Ohno *et al.*, 1997, Mia *et al.*, 1997, Adhvaryu and Erhan, 2002).
- v. The effect of surface's roughness is neglected.
- vi. The chemical content of vegetable oils is not discussed in more detail.



## REFERENCES

- Adhvaryu, A. and Erhan, S. Z. (2002). Epoxidized Soybean Oil as A Potential Source of High-Temperature Lubricants. *Industrial Crops and Products*. 15, 247-254.
- Bahari, A. (2010). Numerical Solution of Elastohydrodynamic Lubrication with Non-Newtonian Fluid Flow. Master of Engineering, Universiti Teknologi Malaysia, Skudai.
- Bannister, K. E. (1996). Lubrication for Industry (1<sup>st</sup>ed.). New York: Industrial Press Inc.
- Bartz, W. J. (1998). Lubricants and the Environment. *Tribology International*. 31 (1-3), 35-47.
- Biresaw, G. (2006). Elastohydrodynamic Properties of Seed Oils. *Journal of the American Oil Chemists' Society (JAOC)*. 83(6), 559-566.
- Bisht, R. P. S., Sivasankaran, G. A. and Bhatia, V. K. (1993). Additive Properties of Jojoba Oil for Lubricating Oil Formulations. *Wear*. 161, 193-197.
- Blahey, A., G. and Schneider, G., E. (1987). A Numerical Solution of the Elastohydrodynamic Lubrication of Elliptical Contacts with Thermal Effects. *Tribology Series*. 11, 219-230.

- Bremmer, B. J. and Plonsker, L. (2008). *Bio-Based Lubricants: A Market Opportunity Study Update. Prepared for the United Soybean Board*. Chesterfield: Omni Tech International, Ltd.
- Chapra, S. C. (2008). *Applied Numerical Methods with MATLAB for Engineers and Scientists*. (2<sup>nd</sup>ed.). New York: McGraw-Hill.
- Chu, L.-M., Hsu, H.-C., Lin, J.-R. and Chang, Y.-P. (2009). Inverse Approach for Calculating Temperature in EHL of Line Contacts. *Tribology International*. 42(8), 1154-1162.
- Cioc, C. A. B. (2004). *An Elastohydrodynamic Lubrication Model for Helicopter High-Speed Transmission Components*. Doctor of Philosophy, The University of Toledo, Ohio.
- Dow, T. A., Stockwell, R. D. and Kannel, J. W. (1987). Thermal Effects in Rolling/Sliding EHD Contacts: Part I – Experimental Measurements of Surface Pressure and Temperature. *Journal of Tribology, Transaction of ASME*. 109(3), 503-510.
- Dowson, D. and Higginson, G. R. (1959). A Numerical Solution of the Elastohydrodynamic Lubrication Problem. *Journal of Mechanical Engineering Science*. 1(1), 6-15.
- Dowson, D. and Higginson, G. R. (1977). *Elastohydrodynamic Lubrication*, SI Edition. New York: Pergamon Press.
- Elsharkawy, A. A. (1997). Magnetic Head-rigid Disk Interface Hydrodynamically Lubricated with a Power-law Fluid. *Wear*. 213(1-2), 47-53.
- Erhan, S. Z. and Asadauskas, S. (2000). Lubricant Basestocks from Vegetable Oils. *International Crops and Products*. 11, 277-282.

- Evan, H. P. and Snidle, R. W. (1981). Inverse Solution of the Reynolds' Equation of Lubrication under Point-Contact elastohydrodynamic Conditions. *Journal of Tribology, Transaction of ASME*. 103, 539-546.
- Fein, R. S. (1997). *High Pressure Viscosity and EHL Pressure-Viscosity Coefficient*. In *Tribology Data Handbook*. (pp. 3-33). Boca Raton, Fla: CRC Press.
- Gohar, R. and Safa, M. M. A. (2010). *Fluid Film Lubrication*. In Rahnejat, H. (Ed.) *Tribology and Dynamics of Engine and Powertrain. Fundamentals, Applications and Future Trends*. (pp. 132-170). Cambridge: Woodhead Publishing Ltd.
- Goodyer, C. E. (2001). *Adaptive Numerical Methods for Elastohydrodynamic Lubrication*. Doctor of Philosophy, School of Computing, The University of Leeds, Leeds.
- Grompone, M. A. (2005). *Sunflower Oil*. In Shahidi, F. (Ed.) *Bailey's Industrial Oil and Fat Products* (6<sup>th</sup> ed.) Vol. 2 (pp. 655-730). John Wiley & Sons.
- Hamrock, B. J. and Anderson, W. J. (1983). Rolling-Element Bearings. *NASA Reference Publication*. 1105, 1-63.
- Hamrock, B. J and Dowson, D. (1976). Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part I – Theoretical Formulation. *Journal of Lubrication Technology, Transaction of ASME*. 98, 223-229.
- Hamrock, B.J. , Pan, P. and Lee, R.-T. (1988). Pressure Spikes in Elastohydrodynamically Lubricated Conjunctions. *Journal of Tribology*. 110(2), 279-284.
- Hamrock, B. J., Schmid, S. R. and Jacobson, B. O. (2004). *Fundamentals of Fluid Film Lubrication*. (2<sup>nd</sup> ed.). New York: Marcel Decker, Inc.
- Hamrock, B. J. and Tripp, J. H. (1984). Numerical Methods and Computers Used in Elastohydrodynamic Lubrication. *Paper presented at the Developments in*

*Numerical and Experimental Method Applied in Tribology: Proceedings of the 10<sup>th</sup> Leeds-Lyon Symposium on Tribology.* 6-9 September 1983. Lyon, 1-19.

Houpert, L. G. and Hamrock, B. J. (1986). Fast Approach for Calculating Film Thicknesses and Pressures in Elastohydrodynamically Lubricated Contacts at High Loads. *Journal of Tribology*. 108(3), 411-420.

Houpert, L. G., Ioannidess, E., Kuypers, J. C. and Tripp, J. (1987). The Effect of the EHD Pressure Spike on Rolling Bearing Fatigue. *Journal of Tribology*. 109(3), 444-450.

Jayadas, N. H., Nair, K. P. and Ajithkumar, G. (2007). Tribological Evaluation of Coconut Oil as an Environment-Friendly Lubricant. *Tribology International*. 350-354.

Jayadas, N. H. and Nair, K. P. (2006). Coconut Oil as Base Oil for Industrial Lubricants-Evaluation and Modification of Thermal, Oxidative and Low Temperature Properties. *Tribology International*. 39, 873-878.

Khan, H., Sinha, P. and Saxena, A. (2009). A Simple Algorithm for Thermo-Elasto-Hydrodynamic Lubrication Problems. *International Journal of Research and Review in Applied Sciences*. 1(3), 265-279.

Khonsari, M. M. and Booser, E. R. (2008). *Applied Tribology. Bearing Design and Lubrication*. (2<sup>nd</sup> ed.). Chichester: John Wiley & Sons Ltd.

Knothe, G. and Steidley, K. R. (2007). Kinematic Viscosity of Biodiesel Components (Fatty Acid Alkyl Esters) and Related Compounds at Low Temperatures. *Fuel*. 86, 2560-2567.

Knothe, G. and Steidley, K. R. (2005). Kinematic Viscosity of Biodiesel Fuel Components and Related Compounds. Influence of Compound Structure and Comparison to Petrodiesel Fuel Components. *Fuel*. 84, 1059-1065.

- Laniado-Jácome, E., Meneses-Alonso, J. and Diaz-López, V. (2010). A Study of sliding between rollers and races in roller bearing with a numerical model for mechanical event simulations. *Tribology International*. 43, 2175-2182.
- Lee, R.-T. and Hamrock B. J. (1989a). Squeeze and Entraining Motion in Nonconformal Line Contact. Part I – Hydrodynamic Lubrication. *Journal of Tribology, Transaction of ASME*. 111(1), 1-7.
- Lee, R.-T. and Hamrock B. J. (1989b). Squeeze and Entraining Motion in Nonconformal Line Contact. Part II – Elastohydrodynamic Lubrication. *Journal of Tribology, Transaction of ASME*. 111(1), 8-16.
- Lee, R.-T. and Hsu, C.-H. (1993). A Fast Method for the Analysis of Thermal-Elastohydrodynamic Lubrication of Rolling/Sliding Line Contacts. *Wear*. 166(1), 107-117.
- Lee, R.-T. and Hsu, C.-H. (1994). Advanced Multilevel Solution for Thermal Elastohydrodynamic Lubrication of Simple Sliding Line Contacts. *Wear*. 171(1-2), 227-237.
- Lubrecht, A. A. (1987). *The Numerical Solution of the Elastohydrodynamic Lubricated Line and Point Contact Problem, Using Multigrid Techniques*. Doctor of Philosophy. University of Twente, Den Haag.
- Mia, S. and Ohno, N. (2010). Prospect of Mustard and Coconut Oil as Environment Friendly Lubricant for Bangladesh. *Proceedings of International Conference on Environmental Aspects of Bangladesh (ICEAB10)*. September 2010. Japan, 120-121.
- Mia, S., Hayashi, S. and Ohno, N. (2007). High Pressure Tribological Behaviour of Vegetable Oils as Lubricant. *Proceedings of the International Conference on Mechanical Engineering 2007*. 29-31 December. Dhaka, Bangladesh, 1-5.

- Musa, J. J. (2009). Evaluation of the Lubricating Properties of Palm Kernel Oil. *Leonardo Electronic Journal of Practices and Technologies*. 14, 107-114.
- Oh, K. P. (1984). The Numerical Solution of Dynamically Loaded Elastohydrodynamic Contact as a Nonlinear Complementary Problem. *Journal of Tribology, Transaction of ASME*. 106(1), 88-95.
- Ohno, N., Shiratake, A., Kuwano, N. and Hirano, F. (1997). Behaviour of Some Vegetable Oils in EHL Contacts. *Proceedings of the 23<sup>rd</sup> Leeds-Lyon Symposium on Tribology held in the Institute of Tribology, Department of Mechanical Engineering 1996*. 10-13 September 1996. Leeds, UK. 243-251.
- Okamura, H. (1982). A Contribution to the Numerical Analysis of Isothermal Elastohydrodynamic Lubrication. *Tribology of Reciprocating Engines: Proceedings of the 9<sup>th</sup> Leeds-Lyon Symposium on Tribology*. Guilford, England. 313-320.
- Picigallo, B. (1996). A Fast Method for the Numerical Solution of Thermal-Elastohydrodynamic Lubrication Problems. *Wear*. 193(1), 56-65.
- Pinkus, O. (1987). The Reynolds Centennial: A Brief History of the Theory of Hydrodynamic Lubrication. *Journal of Tribology*. 109(1), 2-20.
- Quinchia, L. A., Delgado, M. A., Valencia, C., Franco, J. M. and Gallegos, C. (2010). Viscosity Modification of Different Vegetable Oils with Eva Copolymer for Lubricant Applications. *Industrial Crops and Products*. 32, 607-612.
- Rudnick, L. R. and Erhan, S. Z. (2005). Natural Oils as Lubricant. In Rudnick, L. R. (Ed.). *Synthetics, Mineral Oils, and Bio-Based Lubricants. Chemistry and Technology*. Boca Raton: Taylor and Francis Group.
- Rudnick, L. R. (2002). A Comparison of Synthetic and Vegetable Oil Esters for Use in Environmentally Friendly Fluids. In Erhan, S. Z. and Perez, J. M. (Eds.). *Biobased Industrial Fluids and Lubricants*. Illinois: AOCS Press.

- Sadeghi, F. (2010). *Elastohydrodynamic Lubrication*. In Rahnejat, H. (Ed.) *Tribology and Dynamics of Engine and Powertrain. Fundamentals, Applications and Future Trends*. (pp. 171-221). Cambridge: Woodhead Publishing Ltd.
- Sadeghi, F. and Dow T. A. (1987). Thermal Effects in Rolling/Sliding Contacts: Part II – Analysis of Thermal Effects in Fluid Film. *Journal of Tribology, Transaction of ASME*. 109(3), 512-517.
- Safa, M. M. A., Anderson, J. C. and Leather, J. A. (1982). Transducers for Pressure, Temperature and Oil Film Thickness Measurement in Bearings. *Sensors and Actuators*. 3, 119-128.
- Stachowiak, G. W. and Batchelor, A. W. (2006). *Engineering Tribology*. (3<sup>rd</sup> ed.). Burlington: Butterworth-Heinemann.
- Syahrullail, S., Zubil, B. M., Azwadi, C. S. N. and Ridzuan, M. J. M. (2011). Experimental Evaluation of Palm Oil as Lubricant in Cold Forward Extrusion Process. *International Journal of Mechanical Sciences*. 53, 549-555.
- Timoshenko, S. and Goodier, J. N. (1970). *Theory of Elasticity*. (3<sup>rd</sup> ed.) New York: McGraw-Hill.
- Venner, C. H. and Ten Napel, W. E. (1989). Numerical Calculations of Pressure Spike in Elastohydrodynamic Lubrication. *Lub. Sci.* 2-4, 321-335.
- Wan Nik, W. B., Ani, F. N., Masjuki H. H. and Eng Giap, S. G. (2005). Rheology Of Bio-Edible Oils According to Several Rheological Models and Its Potential as Hydraulic Fluid. *Industrial Crops and Products*. 22, 249-255.
- Xiaolan, A. and Haiqing, Y. (1988). A Full Numerical Solution for General Transient Elastohydrodynamic Line Contacts and Its Application. *Wear*. 121, 143-159.